Dynamic Investment Strategies for Swiss Pension Funds

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Abstract

Swiss pension funds have to guarantee a minimum return on the mandatory pension capital on a yearly basis. To be able to pay out the guaranteed rate with 100% certainty, financial theory suggests to follow a pro-cyclical dynamic investment strategy. In reality, Swiss pension funds have followed static investment strategies. In contrast to dynamic strategies, static investment strategies are not influenced by the current risk-taking capacity of the fund.

In this study, we analyze four different dynamic strategies and compare them with a static buy-and-hold strategy. The dynamic strategies analyzed are a classic portfolio insurance with call options, two CPPI-type strategies and a straddle-like investment product called Reverso. The assessment of the dynamic investment strategies is based on a Monte Carlo simulation for a hypothetical pension fund. We apply expected maximum shortfall as risk criterion and expected growth rate of funding ratio as reward criterion.

We show in this study that dynamic strategies can be an attractive alternative to a static investment strategy in some circumstances. The analyzed dynamic investment strategies have the potential to offer a more attractive risk-return spectrum than a static buy-and-hold strategy for the risk and return measures used in this study. For any acceptable level of risk, there is a dynamic strategy that yields a higher expected growth rate of funding ratio. However, dynamic strategies should not be seen as panacea for the issues faced by Swiss pension funds as other factors, such as transaction costs, need to be taken into account when choosing an investment strategy.

Contents

1 Introduction

Today, retirement provisions in the western hemisphere - be it private or publicly organized - face the general problem of longer periods of retirement due to a higher average life expectancy and stagnant time-spans of professional activity. This puts governments and pension funds in a situation of increasing expenditures but decreasing contributions. As a consequence, retirement plans and pension funds, which are based on the funding principle, have to rely more and more on their capital gains and hence on the investment performance that can be achieved on capital markets.

The dependence on capital gains has also increased in the second pillar of the Swiss retirement provisions system. Moreover, for several years, the guaranteed interest rate payable on the mandatory pension capital accumulated in accordance with the Swiss pension law has been higher than the riskless interest rate. In fact, legal parameters used for actuarial calculations in Switzerland do not fully reflect actual circumstances. For instance, the conversion rate with which the individual pension capital is converted to an annual pension annuity does not reflect the increased life expectancy. As a consequence, Swiss pension funds are forced to compensate this shortcoming by higher investment returns. Finally, after the drastic reduction of the minimum interest rate in the years 2002, 2003, and 2004, when the majority of Swiss pension funds had funding ratios near or below 100%, pension funds have been in the spotlight of public interest. Not only the retirement provisions system as a whole is questioned, but also the investment performance of Swiss pension funds in the last years is critically reviewed.

According to pension fund surveys conducted in Switzerland, the majority of funds follows static investment strategies. A static investment strategy implies that the strategic and tactical asset allocation of a fund is only partly influenced by its current risk-taking capacity. In this study we analyze whether dynamic investment strategies might be a realistic alternative to static strategies for Swiss pension funds. For this purpose, different dynamic strategies such as portfolio insurance or constant proportion portfolio insurance (CPPI) are compared to a classic buy-and-hold strategy with respect to investment return and shortfall risk.

The study on dynamic investment strategies for Swiss pension funds is structured as follows: the main issues Swiss pension funds face in the current regulatory environment and their investment behavior is discussed in Section 2. Potential dynamic investment strategies for Swiss pension funds are designed and discussed in Section 3. Section 4 describes the modeling framework. Section 5 provides the results of a Monte Carlo simulation for a model pension funds under different investment strategies. Finally, Section 6 concludes.

2 Situation and Investment Behavior of Swiss Pension Funds

According to the Law of Occupational Pension Plans, Swiss funds have to guarantee a certain minimum return on the mandatory pension capital on a yearly basis. The minimum interest rate is intended to ensure that the members of a pension fund receive a reasonable return on their investments. Such a minimum return is essential in pension systems without free choice of fund and only limited flexibility concerning the investment strategy applied to the pension capital.

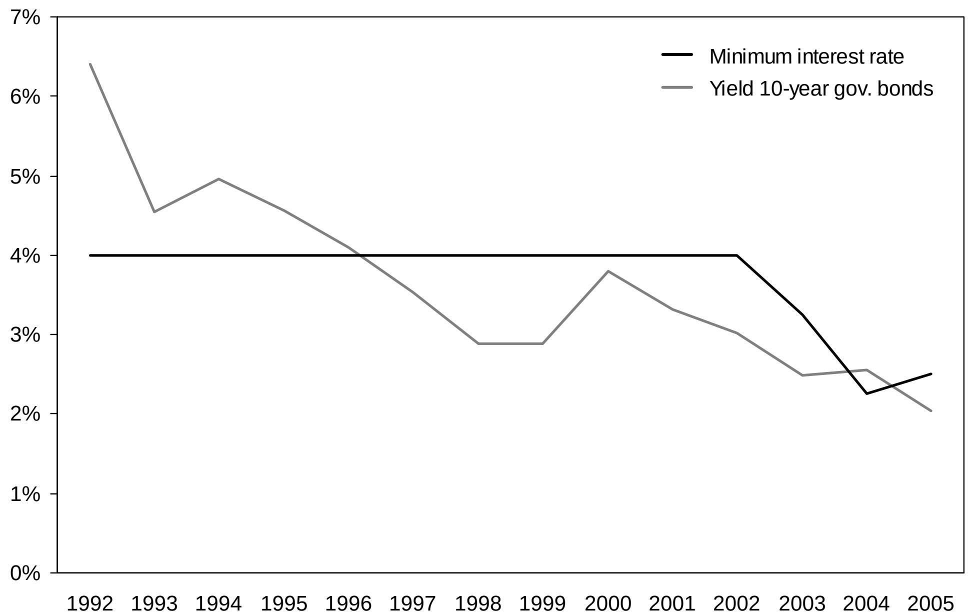

Figure 1: Minimum interest rate and average yields of 10-year Swiss government bonds from 1992 to 2005.

Originally, the minimum interest rate was set to 4%. It was intended to remain at that level for the long run. The funds were presumed to build up reserves from excess returns during the good years and to draw from those reserves during the bad years such that the minimum interest rate would be paid even if it could not be earned on the capital markets. This concept worked as long as interest rates remained above 4%. As shown in Figure 1, the yields of long-term Swiss government bonds were above 4% until the year 1996. During that time, equity exposure was not even necessary for a pension fund to earn the guaranteed rate. When interest rates declined below 4% and stayed there for several years, pension funds were, for some time, still able to pay out 4% because of the rich stock market returns in the years 1996 to 2000. Once the equity returns turned negative in the big stock market slump from 2001 to 2003, it became clear that pension funds would no longer be able to pay out a guaranteed rate of 4% regardless of the market conditions.

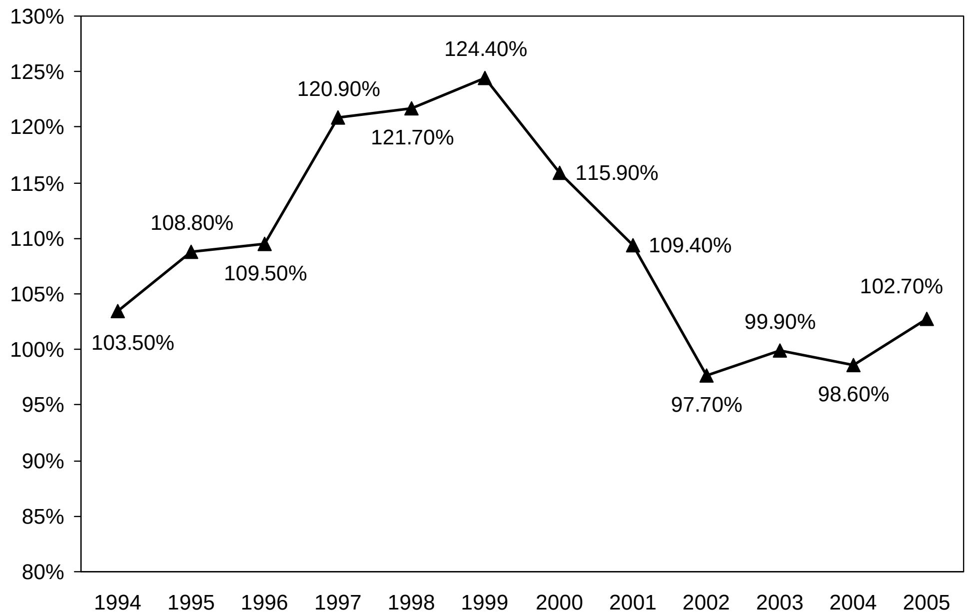

Figure 2: Weighted average of funding ratio of Swiss pension funds from 1994 to 2005 based on a sample of 425 funds with total assets of CHF 324.5 billion (AWP/Complementa "Risiko Check-up").

The combination of historically low interest rates, significantly below the guaranteed rate, and negative stock returns has left its mark. As shown in Figure 2, average funding ratios of Swiss pension funds drastically declined from 124.4% to only 97.7% within only 3 years. As a consequence, the minimum interest rate was, for the first time since the inception of the second pillar in Switzerland, reduced from

4% to 3.25% (2003) and then to 2.25% (2004). After the stock market recovered, the minimum interest rate was increased to 2.5%, effective as of January 2005.

What are the consequences of a system with a minimum interest rate from a portfolio management perspective? To be able to pay out the guaranteed rate with certainty, part of the pension capital has to be invested in riskless fixed income instruments. The remaining capital can be used to generate returns above the minimum interest rate, for example by investing it in stocks. Not surprisingly, a higher funding ratio allows for a larger share invested in risky assets. AMMANN (2003) for instance shows with a simple portfolio insurance model that maximum stock allocations vary from 0% for a funding ratio of 100% to 43% for a funding ratio of 110% when a one year investment horizon is applied.

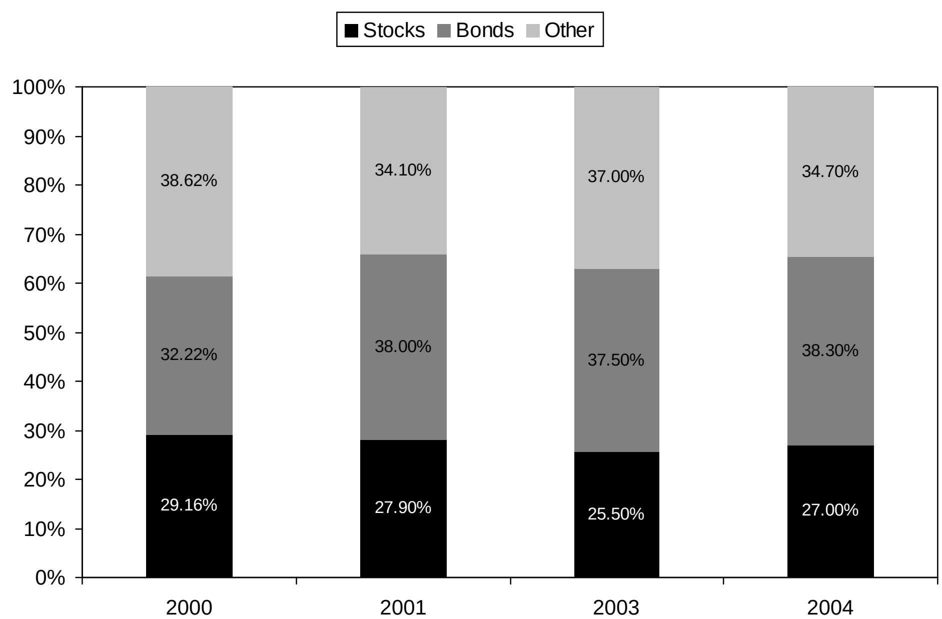

Figure 3: Average effective asset allocation of Swiss pension funds from 2000 to 2004 (Swisscanto pension fund survey).

However, in reality, effective asset allocations including stock holdings of Swiss pension funds remained almost unchanged from 2000 to 2004. Even when the majority of funds had funding ratios near or below 100%, funds did not reduce their stock holdings. It can be concluded that many Swiss pension funds have followed investment strategies that, although attractive with hindsight due to newly booming markets, were not appropriate to guarantee a certain minimum return with 100% certainty. In fact, such a static strategy was only possible because pension funds, unlike insurance companies, were able to implicitly transfer the investment risk to their beneficial owners.

When the minimum interest rate is above the riskless rate, as it has been the case in Switzerland for several years, only over-funded pension funds are able to guarantee the minimum return with 100% certainty. Assume for instance a minimum interest rate of 2.5% and a riskless rate of 1%. To be able to pay out the minimum return of 2.5% in one year with 100% certainty, a pension fund with liabilities of CHF 100 million would have to invest today the present value of CHF 102.5 million discounted at the riskless rate, which comes to CHF 101.5 million. Only a pension fund with assets of at least CHF 101.5 million or a funding ratio of 101.5% is able to invest this amount at the riskless rate. In addition, there is no remaining capital to be invested in riskier assets. However, a stock allocation of zero makes it impossible for a pension fund to earn the minimum return. As a result, the funding ratio of the pension fund in the example would decrease.

Because the regulator accepts a funding ratio temporarily below 100% and no shareholders worry about their equity, unlike in the case of insurance companies, pension funds are able to bear relatively high risk even in the absence of reserves and even if under-funded. In case of a refunding, both employers and employees are required to pay additional funds to bring the funding ratio back to 100%. Additionally, even benefits can be reduced to increase the funding ratio. Hence, pension funds are able to maintain high stock holdings in the absence of reserves because they can transfer the risk associated with such a strategy to the members of the fund. This flexibility is usually highly appreciated by pension fund managers. Economically however, it seems a strange policy to offer return guarantees to the same people to who, at the same time, substantial implicit investments risks are transferred.

3 Dynamic Investment Strategies

Swiss pension funds not only have to guarantee a minimum return on the mandatory pension capital on a yearly basis but also aim at generating investment returns above the guaranteed rate to ensure maximum benefits for the members of the fund. In this section dynamic investment strategies that are consistent with both targets are designed and analyzed.

3.1 Portfolio Insurance

Investment strategies that hedge the value of a portfolio to a certain extent against market risk are referred to as portfolio insurance. To be able to pay out a minimum return with certainty, a part of the pension capital is invested in a riskless fixed income instrument. The amount invested in the riskless fixed income instrument equals the present value of the floor PV_{Floor}, which is liabilities plus the required minimum return:

$$PV_{Floor} = L \cdot \exp\left[(r_{\min} - r_f) \cdot T\right], \tag{1}$$

where L represents the liabilities, $r_{\min}$ the minimum interest rate and r_f the riskless rate. The remaining capital can be invested in risky assets:

$$W_{Risky\ Assets} = A - PV_{Floor}, \tag{2}$$

where A represents total pension assets. To gain maximum upside potential, call options can be used. The return profile of such a strategy is shown in Figure 4.

As Brennan/Schwartz (1976) show, holding a stock position and buying put options is equivalent to holding a riskless investment equal to the present value of the desired floor and buying call options with the remaining capital. In other words, a portfolio insurance strategy with guaranteed minimum return can also be implemented with stocks and put options. However, portfolio insurance with call options offers more degrees of freedom. For portfolio insurance with put options, only one specific exercise price is admissible to guarantee a certain floor. In contrast, for a portfolio insurance with calls, the exercise price is arbitrary because the floor is accomplished by the riskless fixed income investment and not by the option position. In addition, also the number of call options is arbitrary, while for the put strategy the ratio between stocks and options has to be 1:1 to guarantee a stable floor. Finally, both call and put options can be replicated by implementing a dynamic trading strategy in the underlying security so that the use of derivatives is not required to implement a portfolio insurance strategy.

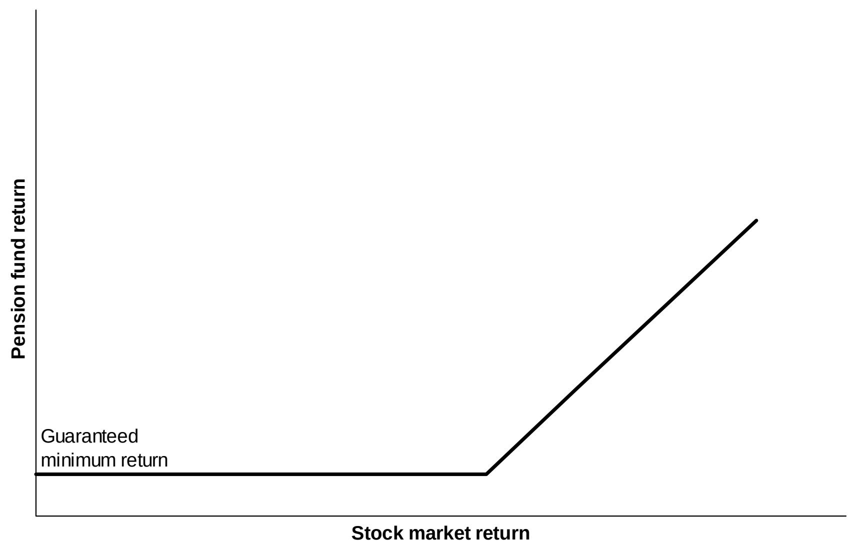

Figure 4: Return profile of a portfolio insurance with guaranteed minimum return.

If the minimum interest rate is below the riskless rate, portfolio insurance strategies will guarantee at least the floor. However, if the minimum interest rate is higher than the riskless rate, as it is currently the case in Switzerland, pension funds are not able to achieve the minimum interest rate without holding stocks. According to equations (1) and (2), in such a scenario only over-funded pension funds have the risk-taking capacity to hold stocks. Assume a pension fund with assets of CHF 100 million and liabilities of also CHF 100 million. According to equation (1), the present value of the floor is higher than CHF 100 million because the minimum interest rate is higher than the riskless rate. As a result, the pension fund in the example is not able to guarantee the minimum rate with certainty. In addition, there is no remaining capital to be invested in stocks, as can be seen from equation (2) and the funding ratio will therefore decrease over time below a value of 100%.

Summarizing, in a scenario with return guarantees above the riskless rate, portfolio insurance works only for over-funded pension funds. Once the surplus is eaten up, portfolio insurance will do poorly and, in fact, will not work anymore.

3.2 Constant Proportion Portfolio Insurance

Another approach to portfolio insurance is known as constant proportion portfolio insurance (CPPI). Because CPPI is not based on option theory, the strategy is simpler to implement than a classic portfolio insurance. CPPI was developed by Black/Jones (1987). They show the application of the CPPI approach to corporate pension funds in Black/Jones (1988).

To implement a CPPI strategy, an investor first selects a floor below which he does not want the value of its portfolio to fall. If we think of the difference between the current portfolio value A and the floor F as a "cushion" C, this amount can basically be invested in risky assets, e.g., in stocks:

$$C = A - F. \tag{3}$$

For example, the floor could be the liabilities or the present value of the sum of the liabilities plus the minimum return guarantee. Especially for a pension fund, the cushion is normally rather small. As a result, only a small fraction of the portfolio can be invested in stocks. To increase the equity exposure, E, the cushion is multiplied with a constant factor $m > 1$:

$$E = m \cdot C. \tag{4}$$

The CPPI decision rule is simply to keep the exposure to risky assets at the constant multiple m of the cushion. As the cushion increases, stocks are bought to capture more of their appreciation potential. As the cushion decreases, risky assets are sold to reduce the exposure. This will keep the portfolio from falling below the floor. To illustrate the CPPI decision rule, assume a pension fund with assets of CHF 110 million, liabilities (floor) of CHF 100 million and a multiplier of 2. Because the initial cushion is CHF 10 million, the initial investment in stocks is CHF 20 million, equally twice the cushion. The remaining capital of CHF 90 million is invested in a riskless asset, e.g., the money market account. Now imagine that the portfolio value falls from CHF 110 to CHF 105 million. The cushion will now equal CHF 5 million. According to the CPPI rule, the new stock position is CHF 10 million. This requires the sale of stocks and the investment of the proceeds in the riskless asset. Now assume that the portfolio value increases from CHF 105 million to CHF 115 million. With a cushion of CHF 15 million, the pension fund can increase its stock position to CHF 30 million, while the position in the riskless asset is reduced to CHF 85 million. As can be seen, following a CPPI strategy means selling stocks as they fall and buying stocks as they rise.

Under a CPPI strategy, the portfolio will do at least as well as the floor. The only scenario in which the portfolio might do worse than the floor is when stock market drops immediately before the investor has had the chance to rebalance. More generally, the market can fall by as much as $1/m$ with no rebalancing before the floor is endangered.

In a bull market, a CPPI strategy will work very well. The strategy requires buying stocks as they rise, with each marginal purchase paying off. However, in flat markets with high stock market volatilities the CPPI strategy will do relatively poorly, because reversals hurt such a strategy. In those situations, stocks are sold on weakness only to see the market rebound and stocks are bought on strength only to see the market weaken. For pension funds, an additional problem occurs when the guaranteed rate is higher than the riskless interest rate as it is currently the case in Switzerland. To invest the non-equity part of the portfolio at the riskless rate would be a suboptimal strategy because the funding ratio will decrease when the difference between the riskless rate and minimum interest rate cannot be compensated by the equity position. Consequently, the non-equity part of the portfolio is invested in (long-term) bonds. As a result, not only the "risky" part of the portfolio fluctuates in value, but also the "riskless" part of the portfolio, implying that the floor can be endangered.

3.3 Reverso

For pension funds, an investment strategy that generates both profits in bull and in bear markets seems to be of special interest. Such a strategy implies a combination of call and put options. One popular strategy with such a profit pattern is a so-called straddle, which involves buying a call and a put with the same strike price and expiration date. The profit profile of a straddle is shown in Figure 5.

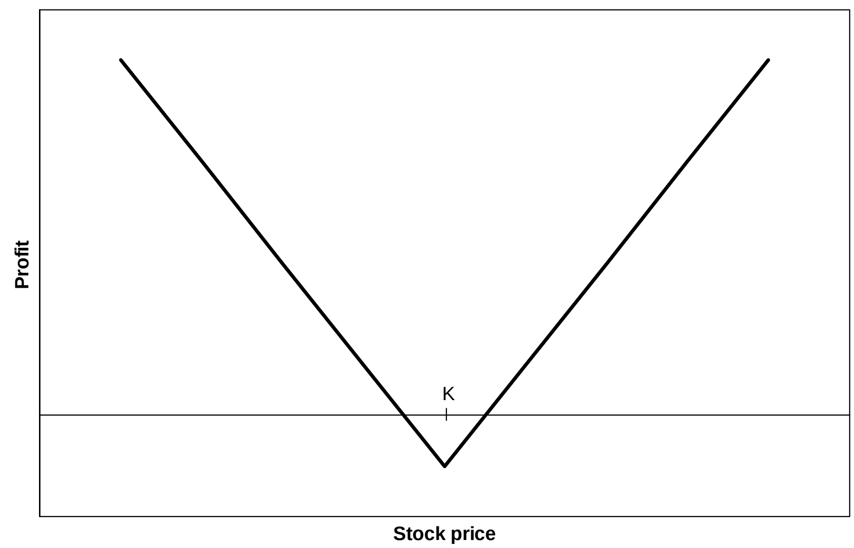

Figure 5: Profit profile of a straddle.

The strike price is denoted by K. If the stock price is close to this strike price at maturity of both options, the straddle leads to a loss. However, if there is a sufficiently large move of the stock price in either direction, a profit will result.

Such straddle-like investment products are offered by investment banks, for example, Société Générale offers a product with such a payoff profile called Reverso. Basically the Reverso strategy corresponds to a straddle. However, the payoff of the Reverso strategy depends not exclusively on the price of the underlying at maturity, but on the maximum and minimum end-of-year prices of the underlying during the lifetime of the product. To illustrate this lock-in feature, assume the end-of-year prices of the Reverso underlying shown in Table 1.

Table 1: Two price path scenarios for the Reverso underlying. A is a bull market scenario, while B is a bear market scenario.

		Time (years)				
		0	1	2	3	4
Scenario A	End-of-year price	100	90	120	150	130
Scenario B	End-of-year price	100	105	90	70	85

The initial price of the Reverso underlying is assumed to be CHF 100, which also equals the exercise price of both the Reverso call and the Reverso put option. In the bull market scenario A, the maximum end-of-year price of the underlying reached during the lifetime of the product equals CHF 150, while the lowest end-of-year price is CHF 90. At maturity the payoff of the Reverso call option is CHF 50 (CHF 150 minus the strike price of CHF 100), while the payoff of the Reverso put option equals CHF 10. However, the Reverso strategy only pays out the higher of the two option payoffs. Thus for scenario A, the Reverso payoff equals CHF 50. In the bear market scenario B, the highest end-of-year price of the underlying is CHF 105 and the minimum end-of-year price of the underlying is CHF 70. Now the payoff of the strategy equals the payoff of the Reverso put option, which is CHF 30 (CHF 100 minus CHF 70). Formally, the payoff of the Reverso product equals:

$$PF_{\text{Reverso}} = Max\left(PF_{\text{Call}}; PF_{\text{Put}}\right) \tag{5}$$

$$PF_{\text{Call}} = Max\left(S_t - S_0; S_{t+1} - S_0; ...; S_T - S_0; 0\right) \tag{6}$$

$$PF_{\text{Put}} = Max\left(S_0 - S_t; S_0 - S_{t+1}; ...; S_0 - S_T; 0\right), \tag{7}$$

where S_t represents the end-of-year price of the Reverso underlying at time t. The lifetime of the Reverso product equals $(T - t)$ years and the exercise price of both the Reverso call and put option equals the price of the underlying at time 0, S_0. In this case, the underlying of the Reverso strategy is a stock basket consisting of 12 equally weighted stocks[1]. The value of one Reverso unit at time zero is determined by a Monte Carlo simulation[2].

Implementing a Reverso strategy does not require rebalancing of assets during the lifetime of the product. This positively effects transaction costs. In contrast, portfolio insurance and CPPI strategies require regular rebalancing to ensure a stable floor. Thus, transaction costs associated with a Reverso strategy tend to be lower than for the two other dynamic strategies.

A Reverso strategy does well in a market environment of high and increasing volatilities. High fluctuations in value of the underlying stock basket in either direction yield high maximum and low minimum prices and therefore higher payoffs of the Reverso product. However, in a market environment with low or decreasing stock market volatility, a Reverso strategy will do poorly. If both the maximum and the minimum price of the Reverso basket during the lifetime of the product are close to the strike price, the Reverso strategy leads to a significant loss. Theoretically even a total loss could occur. Implementation of a Reverso strategy is therefore only justifiable for a limited part of the pension capital, e.g., 10% or 20%.

[1]The Reverso basket of Société Générale consists of the following 12 stocks: Banco Bilbao Vizcaya Argentaria, Canon, Coca-Cola, Electrolux, Eli Lilly, ENI, Kellogg, Nissan Motor, Procter & Gamble, Stora Enso, Telefonica and Verizon.

[2]To determine the value of one Reverso unit during the simulation, the following approximation is applied:

$$P_{\text{Reverso,t}} = Max_t\,(PF_{\text{Call}}; PF_{\text{Put}}) + 1.112 \cdot BS_{Call}(X^C_{L,t}) + 1.112 \cdot BS_{Put}(X^P_{L,t}),$$

where $Max_t\,(PF_{\text{Call}}; PF_{\text{Put}})$ represents the intrinsic value at time t, $BS_{Call}(\cdot)$ the standard Black-Scholes pricing formula for European call options and $BS_{Put}(\cdot)$ the corresponding formula for European put options (see Hull (2003) for more details on the Black-Scholes option pricing formula).

4 Modeling Framework

In this section we describe the modeling framework of the analysis. The assessment of the dynamic investment strategies described in the previous section is based on a Monte Carlo simulation for a hypothetical model pension fund. For simplification, we model a defined-contribution pension fund. The simulation is based on a closed pension fund scheme. This means that we will model the developments of the current fund, without predicting any new entrants. Technical risks such as changes in mortalities or age structure have only a minor influence on the outcome of the analysis. Therefore they will not be considered in the modeling framework. For the same reason, the model does not consider pension administration costs. Because asset management and transaction costs are extremely diverse across different pension funds and difficult to estimate, the model does also not contain these costs.

The investment universe of the model pension fund is limited to Swiss government bonds bearing very limited risk, a well-diversified international equity portfolio as a proxy for the market portfolio and options on the international equity portfolio and on the Reverso basket. Changes in interest rates do not directly influence the value of liabilities, because in Switzerland a so-called "technical rate" independent from market rates is applied to determine the present value of liabilities. However, term structure movements may influence the level of the minimum interest rate. In sum, the modeling framework covers the following risk factors: investment risk associated with bonds and equities for assets and interest rate risk associated with the minimum interest rate on the liability side.

4.1 Assets

Stock prices follow a geometric Brownian motion:

$$dS = \mu S dt + \sigma S dW, \tag{8}$$

where S represents the stock price, μ the expected rate of stock return, σ the volatility of stock prices and dW a Wiener Process[3]. As shown in Table 2, for the simulation of stock returns we assume an expected return of 7.1% and a stock market volatility of 17.9% for the international equity portfolio[4]. For the Reverso basket an expected return of 8.0% and a volatility of 17.7% is assumed[5]. In addition, we run a simulation for the Reverso strategy based on the international equity portfolio to ensure comparability.

Table 2: Input parameter for simulation of stock returns.

	Expected return (μ)	Volatility (σ)
International stock portfolio	7.1%	17.9%
Reverso basket	8.0%	17.7%

Interest rates are modelled according to Cox/Ingersoll/Ross (1985). In contrast to other popular term structure models, interest rates in their model are always non-negative. The process for the short rate r is:

$$dr = a(b - r)dt + \sigma_r \sqrt{r} dW, \tag{9}$$

where a represents the reversion parameter, b the long-term mean of the short rate and σ_r the volatility of the short rate. The Cox/Ingersoll/Ross (1985) term structure model incorporates mean-reversion: the short rate is pulled to the long-term mean b at a rate a.

As shown in Table 3, we assume for the simulation of short rates a reversion parameter of 25%, a long-term mean of 1.80% and a short rate volatility of 1.17% [6]. The simulation starts with an initial value of 0.98% for the short rate[7].

[3] A Wiener process is a stochastic process where the change in a variable during each short period of time of length Δt has a normal distribution with a mean equal to zero and a variance equal to Δt.

[4] Continuous annual return and volatility of the MSCI World Total Return Index in Swiss Franc in the reference period 1994 to 2005.

[5] Continuous annual return and volatility of the Reverso basket consisting of the 12 equally weighted Reverso stocks in the reference period 1994 to 2005.

[6] Based on the continuous Swiss Euromarket 3-month rate with reference period 1994 to 2005.

[7] Corresponds to the Swiss Euromarket 3-month rate as of December 31, 2005.

Table 3: Input parameter for the simulation of short rates.

	Reversion parameter (a)	Long-term mean (b)	Volatility (σ_r)
Short rate	25%	1.80%	1.17%

In contrast to the original model of COX/INGERSOLL/ROSS (1985), where the value of the short rate determines the level of the term structure at time t, we derive the term structure by adding constant term spreads to the simulated short rate. This is a rough approximation, although inconsistent with COX/INGERSOLL/ROSS (1985), it ensures that the current term structure is reflected in the modelled interest rates. The term spreads shown in Table 4 equal the arithmetic average of the difference between the continuous zero coupon rates and the continuous Euromarket 3-month rate in the year 2005. To illustrate the determination of the term structure, assume a short rate of 1.2%. The 10-year zero coupon rate r(10) is determined by adding the corresponding term spread of 1.64% to the simulated short rate, so that the 10-year zero coupon rate corresponds to 2.84%.

Table 4: Average spreads between monthly observed zero coupon rates and Euromarket 3-month short rates in the year 2005.

Term spreads									
r(1)	r(2)	r(3)	r(4)	r(5)	r(6)	r(7)	r(8)	r(9)	r(10)
.25%	.54%	.77%	.95%	1.10%	1.24%	1.36%	1.46%	1.56%	1.64%

The bond portfolio of our model pension fund is assumed to have a constant duration of 10 years during the whole simulation period. Returns of the bond portfolio are determined by the constant duration index (CDI) of Société Générale:

$$CDI_{t+1} = CDI_t + \frac{1}{12} s(t,10) - D\left[s(t+1,10) - s(t,10)\right], \tag{10}$$

where $s(t,10)$ is the 10-year swap rate at time t and D is a constant calibration factor. Swap rates are based on continuous zero coupon rates simulated with the COX/INGERSOLL/ROSS (1985) term structure model. For the calibration factor, a value of 8 is assumed. It becomes evident from equation (10) that the CDI both considers effects of interest rate shifts on the value of the bond portfolio as well as the income from periodic coupon payments. Finally, the valuation of options is based on the Black-Scholes option pricing model.

Input parameters for simulation of asset returns are estimates based on the reference period 1994 to 2005. It has to be kept in mind that the outcome of the simulation is sensitive to these input parameters. For instance, expected returns and volatilities of stocks influence the developing of the simulated price paths, whereas initial option prices, such as the value of one Reverso unit, are influenced above all by the volatility estimate. To get a better understanding how sensitive the simulation results are to changes in parameter values, we conduct a sensitivity analysis in Section 6.

4.2 Liabilities

Swiss Law of Occupational Pension Plans obliges Swiss pension funds to guarantee and to credit to the mandatory pension capital of the insured a certain minimum interest rate on a yearly basis. The Swiss government has the legal competence to define the level of this minimum interest rate. Thereby, actual returns on standard investment categories have to be considered. The law specifies Swiss government bonds, but also equities, corporate bonds and real estate[8].

Although the Swiss Law of Occupational Pension Plans specifies not only government bonds as a reference for the minimum interest rate, since the year 2002 the yields of long-term Swiss government bonds have served as main benchmark. Therefore, in our modeling framework, the minimum interest rate is linked to the 10-year zero coupon rates as a proxy for the yields of long-term Swiss government bonds. The minimum interest rate in the simulation model is assumed to follow the arithmetic average of the previous year's monthly observed continuous 10-year zero coupon rate. Thus the typical time lags, immanent to the current legislative regulation in case of interest rate movements, will be incorporated in the simulation.

[8] Article 15 of the Swiss Law of Occupational Pension Plans.

The value of liabilities at month m in the year y is defined by the following recursive equation:

$$\begin{aligned} L(m, y) &= L(m-1, y) + \frac{1}{12} \cdot L(12, y-1) \cdot (\exp(r_L) - 1) \\ m &= 1, 2, ..., 12 \end{aligned} \tag{11}$$

where L represents the value of liabilities and r_L the continuous minimum interest rate. For instance, if the value of liabilities is CHF 100 million as of December 2005, ($L(12, 2005)$), and the continuous minimum interest rate for the year 2006 is assumed to be 3%, the value of liabilities corresponds to CHF 103.05 million as of December 2006. The value of the liabilities in each month of the year 2006 is then determined by adding one twelfth of CHF 3.05 million to the previous month's value of liabilities.

4.3 Monte Carlo Simulation

To simulate the monthly development of both assets and liabilities, we apply a Monte Carlo Simulation[9] with 5,000 trajectories. The simulation horizon equals 120 months or 10 years. An initial funding ratio of 110% is assumed for the model pension fund.

To simulate realistically correlated processes for stock and bond prices, correlated random numbers have to be generated as input parameters for the Brownian motions[10] as well as for the Cox/Ingersoll/Ross term structure model. Assume two uncorrelated samples X_1 and X_2 from standard normal distributions. By the following linear combination, two correlated samples Y_1 and Y_2 are obtained:

$$Y_1 = X_1 \tag{12}$$

$$Y_2 = \rho X_1 + \sqrt{1-\rho^2} X_2, \tag{13}$$

where ρ is the desired correlation between Y_1 and Y_2. This basic principle can also

[9] A Monte Carlo simulation of a stochastic process is a procedure for sampling random outcomes for the process.

[10] One Brownian motion for the international stock portfolio and a second Brownian motion for the Reverso stock basket.

be applied for n correlated random numbers. According to a procedure known as Cholesky decomposition[11], the covariance matrix Σ is decomposed into the product of two matrices such that:

$$\Sigma = AA^T, \tag{14}$$

where A^T is the transpose of matrix A. Thus the problem of generating correlated random numbers reduces to finding a matrix A such that $AA^T = \Sigma$. The following expressions make possible a simple recursion to find the elements of matrix A:

$$a_{ii} = \sqrt{\Sigma_{ii} - \sum_{k=1}^{i-1} a_{ik}^2} \tag{15}$$

$$a_{ji} = \frac{1}{a_{ii}} \left(\Sigma_{ji} - \sum_{k=1}^{j-1} a_{ik} a_{jk} \right) \quad j < i. \tag{16}$$

Finally, correlated random numbers are obtained by the following linear combination:

$$Y = AX. \tag{17}$$

Table 5 shows the correlations between short rate changes, returns on the international stock portfolio and returns on the Reverso basket used for the simulation. The correlations are based on the Euromarket 3-month rate, the returns of the MSCI World Index and the returns of the Reverso basket in the reference period 1994 to 2005.

Table 5: Correlation between changes of short rate, international stock portfolio (MSCI World) and the Reverso basket in the reference period 1994-2005.

	Short rate (dr)	Stocks (MSCI)	Stocks (Reverso)
Short rate (dr)	1.00	0.15	0.14
Stocks (MSCI)	0.15	1.00	0.87
Stocks (Reverso)	0.14	0.87	1.00

[11] See Glasserman (2004) for a more detailed description of the Cholesky factorization.

4.4 Risk and Reward Criteria

A fair assessment of different investment strategies requires not only a reward criterion but also a risk criterion. Because Swiss pension fund regulation imposes a so-called "safety principle" on funds, which states that the funding ratio must not fall below 100%, the risk criterion should reflect the shortfall risk of an investment strategy. There are several concepts to quantify the shortfall risk. On the one hand, there are risk measures that quantify the shortfall risk at the end of the simulation period. For instance, the shortfall probability equals the probability of the funding ratio to be below 100% at the end of the simulation by relating the number of simulation trajectories with an end value of the funding ratio below 100% to the total number of trajectories. The expected shortfall is another risk criterion that quantifies the magnitude of shortfall occurrences at the end of the simulation period. The expected value of shortfall is determined as the average of shortfalls across the simulation trajectories. Finally, the conditional expected shortfall is determined as the average of shortfalls only for trajectories with a funding ratio below 100% at the end of the simulation period.

In spite of the "safety principle", the regulator tolerates a funding ratio below 100% temporarily. However, it is generally agreed that if the funding ratio decreases below 90%, restructuring measures such as additional payments of active members of the fund are required. Consequently, the risk criterion should not only reflect the shortfall risk at the end of the simulation period but also during the simulation period. We therefore propose to use the expected maximum shortfall. To determine the expected maximum shortfall for a specific investment strategy, first the maximum shortfall is determined for each simulation trajectory:

$$\begin{aligned} SF_{\max,n} &= Max_n \left(1 - \frac{A_1}{L_1}; ...; 1 - \frac{A_{120}}{L_{120}}; 0\right) \\ with\ n &= 1, 2, ..., 5,000, \end{aligned} \tag{18}$$

where A represents the monthly observed value of assets and L the monthly observed value of liabilities. In a second step, the expected maximum shortfall is determined as the average of the maximum shortfalls across the 5,000 trajectories of the simulation:

$$E\left(SF_{\max}\right) = \frac{1}{5,000}\left(SF_{\max,1}, ..., SF_{\max,5,000}\right). \tag{19}$$

Expected maximum shortfall only considers the mean of the shortfall distribution. To quantify the magnitude of extreme shortfalls that might be encountered in the tail of the distribution, an additional risk measure is used. We refer to this risk measure as highest maximum shortfall. For some value of the confidence parameter α, the highest maximum shortfall is defined as $(1-\alpha)$ quantile of the shortfalls experienced over some period of time. For instance, the 95% highest maximum shortfall is the 5% quantile of the shortfalls during the considered time interval.

Looking at the shortfall risk is only one side of the story. A pension fund also aims at maximizing the benefits of its members. Therefore we apply as reward criterion the expected annual growth rate of the funding ratio to assess different investment strategies. In a first step, the annual growth rate of the funding ratio is determined for each trajectory:

$$\begin{aligned} AGR_{FR,n} &= \frac{1}{10}\left(\ln\left(\frac{A_{120,n}}{L_{120,n}}\right) - \ln(1.1)\right). \\ n &= 1, 2, ..., 5000 \end{aligned} \tag{20}$$

Finally, the expected annual growth rate of the funding ratio corresponds to the average of equation (20) across the 5,000 trajectories of the Monte Carlo simulation:

$$E\left(AGR_{FR}\right) = \frac{1}{5,000}\left(AGR_{FR,1}, ..., AGR_{FR,5,000}\right). \tag{21}$$

An $E\left(AGR_{FR}\right) > 0$ means that asset are expected to grow faster than liabilities, whereas an $E\left(AGR_{FR}\right) < 0$ is synonymous to a funding ratio expected to decrease.

As for the risk criterion, there are also different concepts to measure the reward of an investment strategy. It has to be kept in mind that the outcome of the simulation is sensitive to the selection of both the risk and the reward criterion.

5 Simulation Results

In this section the results of the simulations for the model pension fund under different investment strategies are described. A classical buy-and-hold strategy serves as benchmark for the dynamic strategies.

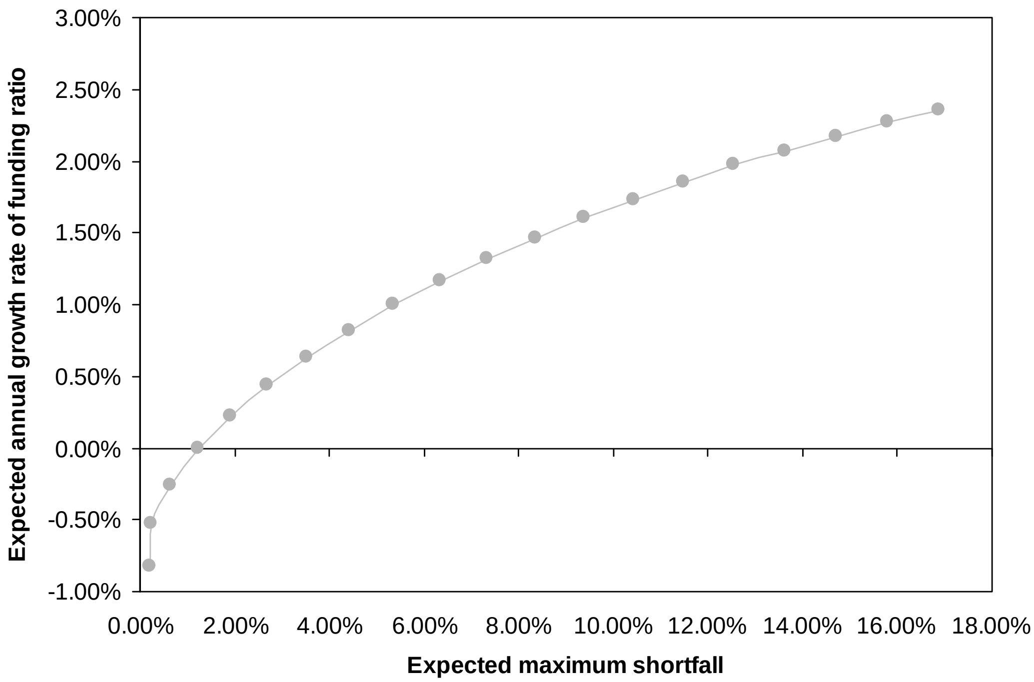

Figure 6: Expected annual growth rate of funding ratio and expected maximum shortfall of a buy-and-hold strategy.

A buy-and-hold strategy is characterized by an initial portfolio allocation (e.g., 60% stocks and 40% bonds) that is bought and then held. Thus, no reallocations are conducted during the whole simulation period of 120 months. Figure 6 shows the expected annual growth rate of the funding ratio $E(AGR_{FR})$ and the expected maximum shortfall $E(SF_{\max})$ for different initial portfolio allocations. Table 6 presents the initial allocations in stocks and bonds and the numerical results of the simulation.

For instance, allocation 7 implies an initial equity allocation of 30%, while the remaining capital, 70%, is invested in bonds. Assuming no reallocations to the initial portfolio weights during the simulation, the funding ratio of a pension fund with allocation 7 is expected to grow by 0.63% per year. The maximum shortfall is expected to be not greater than 3.53%, implying an expected minimum funding ratio of 96.47% across the simulation horizon of 120 months. Not surprisingly, the expected annual growth rate of the funding ratio $E\left(AGR_{FR}\right)$ and expected maximum shortfall $E\left(SF_{\max}\right)$ are increasing with the initial equity share.

Table 6: Asset allocation, expected annual growth rate of funding ratio and expected maximum shortfall of the buy-and-hold benchmark strategy.

Allocation	Stocks	Bonds	$E\left(AGR_{FR}\right)$	$E\left(SF_{\max}\right)$
1	0%	100%	-0.83%	0.21%
2	5%	95%	-0.53%	0.24%
3	10%	90%	-0.26%	0.65%
4	15%	85%	-0.01%	1.23%
5	20%	80%	0.22%	1.91%
6	25%	75%	0.44%	2.69%
7	30%	70%	0.63%	3.53%
8	35%	65%	0.82%	4.43%
9	40%	60%	1.00%	5.37%
10	45%	55%	1.16%	6.34%
11	50%	50%	1.32%	7.34%
12	55%	45%	1.46%	8.36%
13	60%	40%	1.60%	9.39%
14	65%	35%	1.73%	10.44%
15	70%	30%	1.85%	11.49%
16	75%	25%	1.97%	12.56%
17	80%	20%	2.07%	13.64%
18	85%	15%	2.17%	14.72%
19	90%	10%	2.27%	15.81%
20	95%	5%	2.35%	16.90%

Which of the 20 asset allocations presented in Table 6 are appropriate for our model pension fund? In general, allocations with a negative $E\left(AGR_{FR}\right)$ are not acceptable because they sooner or later lead to an under-funding of the pension fund. If asset management and pension administration costs are considered, an allocation with an $E\left(AGR_{FR}\right)$ of at least 0.6%[12] has to be selected to maintain the current funding ratio in the long-run. On the risk side, the regulator temporarily tolerates under-funding to a level of 90%. However, funding ratios at and below 90% usually require restructuring measures such as additional payments of both employers and employees or even reduction of benefits. Therefore an allocation with an $E\left(SF_{\max}\right)$ of more than 10% does not seem to be acceptable for a Swiss pension fund. Consequently, only allocations 7 to 13 are appropriate under current pension fund regulation. Finally, the decision for or against one of the allocations 7 to 13 depends on the individual risk tolerance of the fund.

5.1 Portfolio Insurance

The first dynamic strategy compared to the buy-and-hold benchmark strategy is a classic portfolio insurance with call options. At the end of each year during the simulation, the discounted value of the floor amount, which is the current value of liabilities plus the required minimum return on the liabilities, is invested at the 1-year zero coupon rate. Between 5% (allocation 1) and 100% (allocation 20) of the remaining capital is invested in call options[13]. We refer to the remaining capital as cushion in this section.

We simulate both portfolio insurance with at-the-money and with out-of-the-money call options. As AMMANN/ZIMMERMANN (1998) show, portfolio insurance with at-the-money call options benefits already from small upward movements of the stock market, while participation with out-of-the money calls starts later but turns out be higher. Figure 7 shows expected annual growth rate of the funding ratio $E\left(AGR_{FR}\right)$ and expected maximum shortfall $E\left(SF_{\max}\right)$ of portfolio insurance with at-the-money and with out-of-the-money call options[14]. Numerical results and the share of cushion invested in call options is presented in Table 7.

[12] According to the 2005 Swisscanto pension fund survey asset management and pension administration costs account for about 0.6% of assets per year.

[13] If not all of the remaining capital is invested in call options, the residual is invested at the 1-year zero coupon rate.

[14] Out-of-the-money call options assumed to be 5% out-of-the-money.

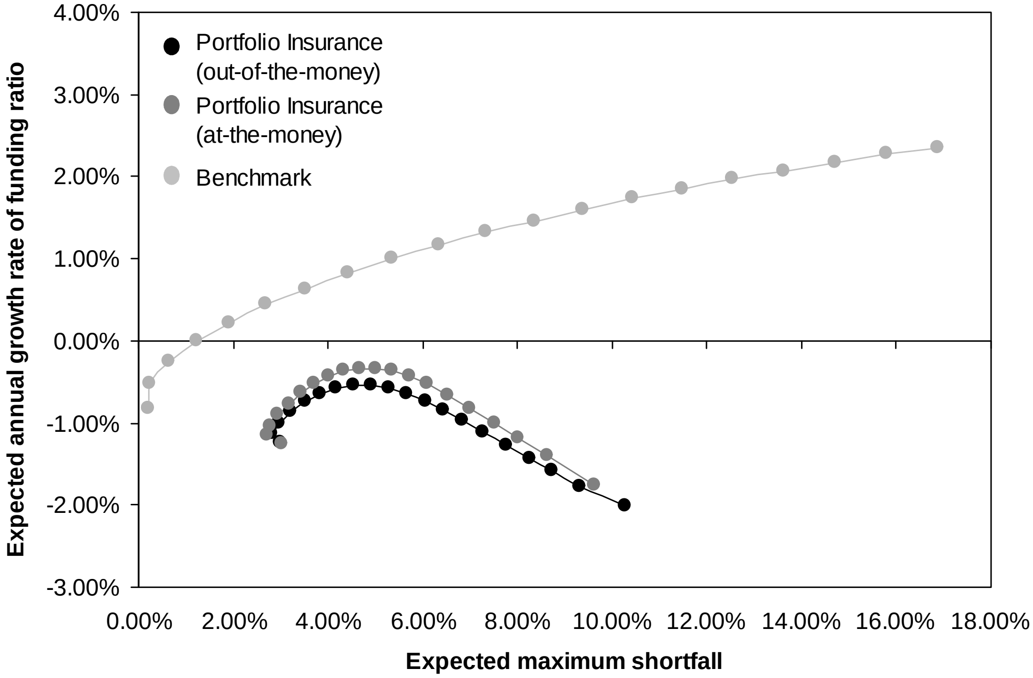

Figure 7: Expected annual growth rate of funding ratio and expected maximum shortfall of portfolio insurance strategy with at-the-money and out-of-the money calls compared to a buy-and-hold strategy. The floor equals the discounted sum of the current value of liabilities and the guaranteed return on the liabilities.

Compared to the buy-and-hold strategy, portfolio insurance with at–the-money and with out-of-the-money call options yields always lower $E(AGR_{FR})$ for any given risk. Portfolio insurance seems not only to be less efficient than the benchmark strategy, but also none of the allocations yields a positive $E(AGR_{FR})$, as shown in Figure 7. To get a better understanding why portfolio insurance yields negative expected growth rates of funding ratio, we have to take into account that the 1-year zero coupon rate in our simulation model is always lower than the minimum interest rate. As outlined before, the minimum interest rate in our modeling framework is the arithmetic average of the previous year's monthly observed 10-year zero coupon rate. Consequently, the average negative difference between the 1-year zero coupon rate and the minimum interest rate of about 1.4%[15] has to be compensated for by the option position to maintain the current funding ratio.

Table 7: Expected annual growth rate of funding ratio and expected maximum shortfall of portfolio insurance with at-the-money and out-of-the-money call options for different allocations. Allocations differ by the share of cushion invested in calls. The cushion equals the difference between assets and the discounted floor amount. The floor equals the discounted sum of liabilities plus the guaranteed minimum return.

		At-the-money calls		Out–of-the-money calls	
Allocation	Calls	$E\left(AGR_{FR}\right)$	$E\left(SF_{\max}\right)$	$E\left(AGR_{FR}\right)$	$E\left(SF_{\max}\right)$
1	5%	-1.26%	3.03%	-1.24%	2.99%
2	10%	-1.16%	2.74%	-1.13%	2.83%
3	15%	-1.04%	2.78%	-1.00%	2.97%
4	20%	-0.90%	2.95%	-0.87%	3.22%
5	25%	-0.77%	3.18%	-0.74%	3.52%
6	30%	-0.63%	3.44%	-0.65%	3.85%
7	35%	-0.52%	3.73%	-0.58%	4.19%
8	40%	-0.43%	4.04%	-0.54%	4.55%
9	45%	-0.37%	4.35%	-0.54%	4.92%
10	50%	-0.34%	4.68%	-0.58%	5.30%
11	55%	-0.34%	5.02%	-0.64%	5.68%
12	60%	-0.37%	5.36%	-0.73%	6.06%
13	65%	-0.43%	5.72%	-0.84%	6.45%
14	70%	-0.53%	6.11%	-0.97%	6.85%
15	75%	-0.67%	6.53%	-1.11%	7.28%
16	80%	-0.83%	7.01%	-1.27%	7.77%
17	85%	-1.00%	7.52%	-1.43%	8.28%
18	90%	-1.18%	8.03%	-1.59%	8.75%
19	95%	-1.40%	8.63%	-1.78%	9.33%
20	100%	-1.76%	9.65%	-2.02%	10.29%

Apparently, the return on the option position is not sufficient to compensate for this negative return difference and the funding ratio is expected to decline. Thus, portfolio insurance with a 100%-floor does not seem to be an efficient strategy for a

[15] Corresponds to the average spread between the 1-year zero coupon rate (0.25%) and the 10-year zero coupon rate (1.64%).

pension fund in the current regulatory environment, if measured by our risk-return framework.

Portfolio insurance with a lower floor might yield better results in such a scenario because more capital is available to compensate for the negative difference between the 1-year zero coupon rate and the minimum interest rate. To test this hypothesis, we assume a new floor at only 90% of the "old" floor amount, which is the discounted sum of the current value of liabilities plus the required minimum return. Figure 8 shows the results of the simulation, numerical results are presented in Table 8.

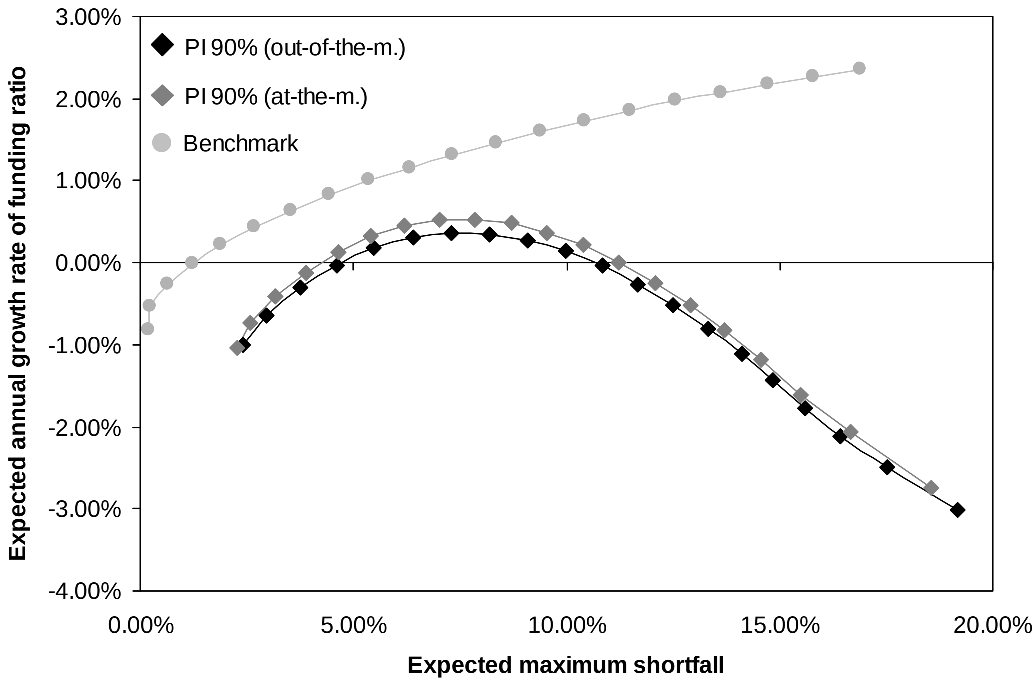

Figure 8: Expected annual growth rate of funding ratio and expected maximum shortfall of portfolio insurance strategy with 90%-floor compared to a buy-and-hold strategy. The floor is the discounted sum of the current value of liabilities plus the guaranteed return on the liabilities.

As presumed, under the current pension fund regulation in Switzerland portfolio insurance with a 90%-floor seems to yield better results than a strategy with a 100%-floor. However, even if the lower floor is applied, portfolio insurance yields only $E(AGR_{FR})$ below the minimum return of about 0.6% necessary to maintain the current funding ratio in the long-run. To invest the floor amount at the riskless rate (1-year zero coupon rate) is therefore a suboptimal strategy because the funding ratio will decrease when the difference between the riskless rate and the minimum interest rate cannot be compensated for by the option position.

Table 8: Expected annual growth rate of funding ratio and expected maximum shortfall of portfolio insurance with at-the-money and out-of-the-money call options for different allocations. The floor equals 90 percent of the discounted sum of liabilities plus the guaranteed minimum return.

		At-the-money calls		Out-of-the-money calls	
Allocation	Calls	$E(AGR_{FR})$	$E(SF_{\max})$	$E(AGR_{FR})$	$E(SF_{\max})$
1	5%	-1.04%	2.26%	-1.00%	2.40%
2	10%	-0.73%	2.58%	-0.65%	2.95%
3	15%	-0.41%	3.18%	-0.31%	3.74%
4	20%	-0.12%	3.88%	-0.03%	4.60%
5	25%	0.13%	4.63%	0.18%	5.49%
6	30%	0.32%	5.41%	0.31%	6.39%
7	35%	0.46%	6.21%	0.37%	7.29%
8	40%	0.52%	7.03%	0.35%	8.20%
9	45%	0.53%	7.86%	0.27%	9.10%
10	50%	0.48%	8.70%	0.14%	9.98%
11	55%	0.37%	9.54%	-0.04%	10.83%
12	60%	0.21%	10.38%	-0.26%	11.67%
13	65%	0.01%	11.22%	-0.52%	12.51%
14	70%	-0.24%	12.08%	-0.80%	13.33%
15	75%	-0.51%	12.90%	-1.11%	14.10%
16	80%	-0.83%	13.70%	-1.43%	14.85%
17	85%	-1.19%	14.55%	-1.77%	15.61%
18	90%	-1.61%	15.49%	-2.12%	16.42%
19	95%	-2.06%	16.67%	-2.50%	17.51%
20	100%	-2.74%	18.55%	-3.01%	19.18%

A potential solution might be to invest the floor amount in long-term bonds. As a result, not only the option position in the portfolio fluctuates in value, but also the "riskless" part of the portfolio, implying that the floor might be endangered. To test such a strategy, we simulate portfolio insurance with at-the-money call options and a 90%-floor. The floor amount is not invested at the riskless rate, but in a bond portfolio with a constant duration of 10 years. Figure 9 shows the results of the simulation, numerical results are presented in Table 9.

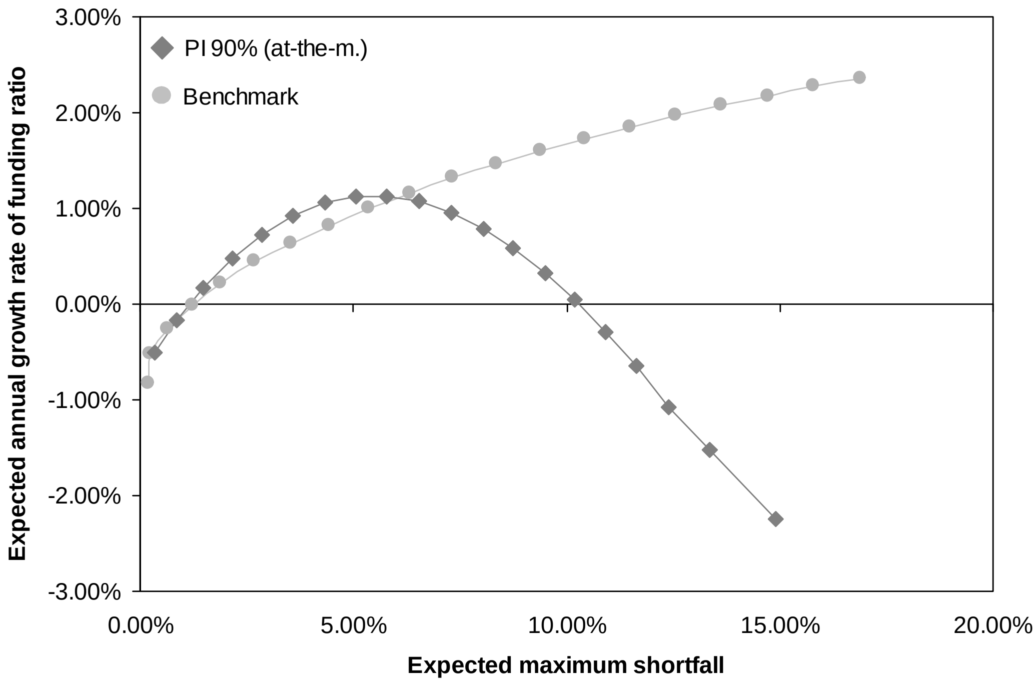

Figure 9: Expected annual growth rate of funding ratio and expected maximum shortfall of portfolio insurance strategy with 90%-floor compared to a buy-and-hold strategy. The floor is invested in bond portfolio with constant duration of 10 years. The floor is the discounted sum of the current value of liabilities plus the guaranteed return on the liabilities.

Allocations 3 to 9 of the tested portfolio insurance yield for the same amount of risk a higher $E(AGR_{FR})$ than the benchmark strategy. If the risk tolerance of the fund allows an $E\left(SF_{\max}\right)$ between 1.48% and 5.79%, a portfolio insurance with 15% to 45% of the cushion invested in at-the-money call options while the remaining capital is invested in long-term bonds seems to be slightly more efficient than following a buy-and-hold strategy. However, if the risk tolerance of the pension fund allows a riskier investment strategy, a buy-and-hold strategy still offers a more attractive risk-return spectrum.

Table 9: Expected annual growth rate of funding ratio and expected maximum shortfall of portfolio insurance with at-the-money call options for different allocations. The floor equals 90 percent of the discounted sum of liabilities plus the guaranteed minimum return and is invested in bond portfolio with constant duration of 10 years.

		At-the-money calls	
Allocation	Calls	$E(AGR_{FR})$	$E(SF_{\max})$
1	5%	-0.51%	0.33%
2	10%	-0.17%	0.85%
3	15%	0.17%	1.48%
4	20%	0.47%	2.16%
5	25%	0.73%	2.86%
6	30%	0.93%	3.59%
7	35%	1.06%	4.33%
8	40%	1.13%	5.06%
9	45%	1.13%	5.79%
10	50%	1.07%	6.55%
11	55%	0.96%	7.30%
12	60%	0.79%	8.04%
13	65%	0.58%	8.76%
14	70%	0.33%	9.49%
15	75%	0.04%	10.20%
16	80%	-0.29%	10.90%
17	85%	-0.65%	11.62%
18	90%	-1.07%	12.40%
19	95%	-1.53%	13.34%
20	100%	-2.25%	14.92%

Theoretically, $E(SF_{\max})$ of a pension fund following a portfolio insurance strategy should be zero. However, under the current regulation with a guaranteed return above the riskless rate, portfolio insurance only provides 100% protection as long as the pension fund is over-funded. This explains why $E(SF_{\max})$ in Table 7, 8 and 9 takes values greater than zero.

Summarizing, under the current regulation in Switzerland, classic portfolio insurance with call options yields less attractive risk-return relations than the benchmark strategy in our modeling framework. The risk-return space appropriate for a Swiss pension fund, $E(AGR_{FR})$ of at least 0.6% and $E\left(SF_{\max}\right)$ below 10% is not even feasible applying a classic portfolio insurance. In a scenario with a minimum interest rate above the riskless rate, portfolio insurance with the floor amount invested in long-term bonds seems to be the better choice. Finally, a floor that permits under-funding but protects the pension fund to a certain extent against the danger of restructuring should be favoured.

5.2 Constant Proportion Portfolio Insurance

In this section we analyze two dynamic strategies based on the CPPI approach. The first dynamic investment strategy corresponds to the classic CPPI approach described in Section 3. We choose a floor equal to the value of liabilities so that the cushion corresponds to the surplus of the pension fund. A multiplier of 2 is assumed for the simulation. Because the minimum interest rate is above the riskless rate, the non-risky part of the portfolio is not invested at the riskless rate, but in a bond portfolio with a constant duration of 10 years. Finally, the portfolio is monthly rebalanced according to the CPPI rule. We refer to this first strategy as CPPI "classic".

The second CPPI-type strategy is based on an investment approach of Société Générale. The approach consists of a self-financing dynamic strategy where always a constant multiple of the net portfolio value is invested in stocks. We assume a multiplier of 3 for the simulation. To be able to invest 3 times the net portfolio value in stocks, an amount equal to two times the net portfolio value has to be borrowed. As we do not consider credit risk in our modeling framework, the model pension fund is able to borrow at the riskless rate. To get a better understanding of the strategy, assume an initial net portfolio value of CHF 10 million, a multiplier of 3 and a riskless rate of 1%. The initial investment in stocks equals CHF 30 million financed by borrowing CHF 20 million. Now imagine that stocks fall from CHF

30 to 28 million within one year. Because the credit is worth -CHF 20.2 (CHF 20 million times 1.01), the net portfolio value equals CHF 7.8 million. The portfolio is rebalanced by reducing the stock position from CHF 28 to 23.4 million (3 times CHF 7.8 million). Thus, also the credit can be reduced to CHF 15.6 million.

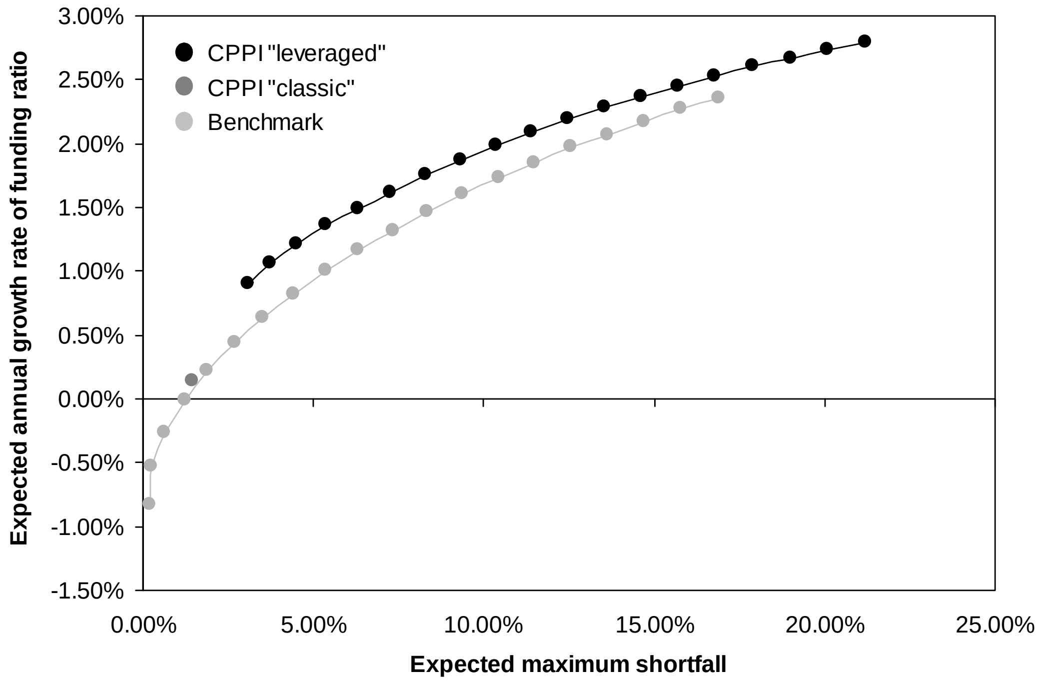

Figure 10: Expected annual growth rate of funding ratio and expected maximum shortfall of CPPI strategies compared to a buy-and-hold strategy. 100% of the pension capital is invested according to the CPPI "classic" strategy (strategy represented only by one point). 10% of the pension capital is invested according to the CPPI "leveraged" approach, the remaining capital is invested in stocks and bonds.

Now assume that stocks increase from CHF 23.4 to CHF 25 million in the second year. With a new net portfolio value of CHF 9.2 million (CHF 25 - CHF15.8 million), the stock position is increased to CHF 27.6 million. It becomes evident that such a strategy implies the typical CPPI approach of selling stocks as they fall and buying stocks as they rise. In the following we refer to the second CPPI-type strategy as CPPI "leveraged". For the simulation we assume monthly rebalancing and a multiplier of 3. Figure 10 shows the simulation results for both CPPI strategies, the numerical results are presented in Table 10.

Table 10: Asset allocation, expected annual growth rate of funding ratio and expected maximum shortfall of CCPI strategies.

Allocation	Stocks	Bonds	CPPI	$E(AGR_{FR})$	$E(SF_{\max})$
1	0%	90%	10%	0.90%	3.09%
2	5%	85%	10%	1.06%	3.75%
3	10%	75%	10%	1.21%	4.53%
4	15%	70%	10%	1.36%	5.39%
5	20%	65%	10%	1.49%	6.32%
6	25%	60%	10%	1.62%	7.29%
7	30%	55%	10%	1.75%	8.29%
8	35%	50%	10%	1.87%	9.32%
9	40%	45%	10%	1.98%	10.35%
10	45%	40%	10%	2.09%	11.41%
11	50%	35%	10%	2.19%	12.47%
12	55%	30%	10%	2.28%	13.54%
13	60%	30%	10%	2.37%	14.62%
14	65%	25%	10%	2.45%	15.71%
15	70%	20%	10%	2.53%	16.80%
16	75%	15%	10%	2.61%	17.90%
17	80%	10%	10%	2.67%	19.00%
18	85%	5%	10%	2.73%	20.10%
19	90%	0%	10%	2.79%	21.21%
20	0%	0%	100%	0.14%	1.48%

CPPI "classic" is applied for total pension capital (allocation 20 in Table 10). That explains why the strategy is only represented by one single point in Figure 10. For an $E(SF_{\max})$ of 1.48%, which implies an expected minimum funding ratio of 98.52%, CPPI "classic" yields an $E(AGR_{FR})$ of 0.14%. Thus, CPPI "classic" yields an $E(AGR_{FR})$ below the minimum return necessary to maintain the current funding ratio in the long-run. Therefore, CPPI "classic" does not seem to be an efficient strategy for a pension fund in the current regulatory environment.

As the name already implies, CPPI "leveraged" is a leveraged investment strategy.

A stock market slump of as much as $1/m$ would destroy as much capital as the total net portfolio value. Therefore, implementation of the CPPI "leveraged" strategy is only justifiable for a limited part of the pension capital. In our simulation 10% of the pension capital is invested according to the CPPI "leveraged" strategy. The remaining capital is invested in stocks and bonds following a buy-and-hold strategy. Initial allocations are given in Table 10.

For any risk above 3.09%, CPPI "leveraged" yields a higher $E(AGR_{FR})$ than the benchmark strategy in our simulation framework. However, for a fund with a very low risk tolerance, a different investment strategy might be the better choice. Furthermore, allocations 9 to 19 imply an $E(SF_{\max})$ of more than 10% which is not an acceptable level of risk for a Swiss pension fund, because the regulator accepts a funding ratio below 100% temporarily, but usually demands restructuring measures in case of a funding at or below 90%. As a result the risk-return spectrum of CPPI "leveraged" is restricted to allocations 1 to 8.

Summarizing, only CPPI "leveraged" seems to offer more attractive risk-return relations than a buy-and-hold strategy. Implementing the CPPI "leveraged" strategy requires a certain minimum risk tolerance of the pension fund.

5.3 Reverso

In contrast to portfolio insurance and CPPI strategies, the Reverso approach offers upside potential both in up and in down markets. Because such a strategy bears a high risk especially in a market environment with low stock market volatility, only a limited part of the pension capital can be invested according to the Reverso approach.

For the simulation we assume that our model pension fund initially invests 10% of its pension capital in the Reverso strategy. The remaining capital is invested in stocks and bonds following a buy-and-hold strategy. Finally, the Reverso lock-in feature refers to the price of the underlying Reverso basket observed at the end of each year during the simulation period. Figure 11 shows the simulation results and compares the Reverso strategy to the buy-and-hold benchmark strategy, whereas Table 11 presents the initial asset allocations and the numerical results of the simulation.

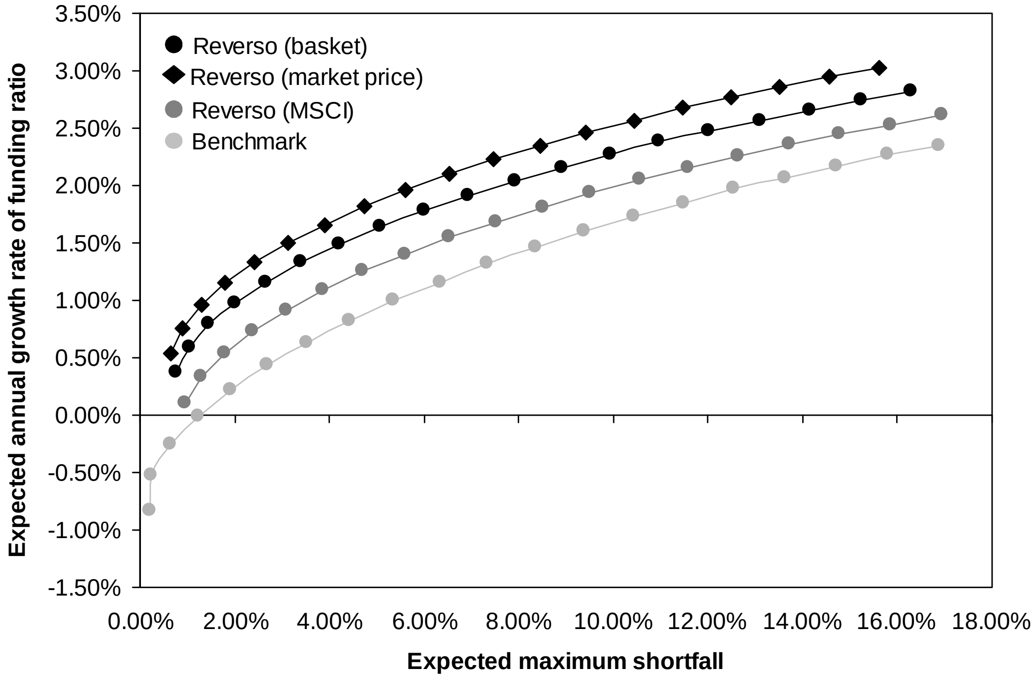

Figure 11: Expected annual growth rate of funding ratio and expected maximum shortfall of Reverso compared to a buy-and-hold strategy. In contrast to the Reverso (basket) product, Reverso (MSCI) is based on a basket consisting of the MSCI World and the price of the Reverso (market price) is a quoted and not a simulated price.

For any risk above 0.76%, the Reverso strategy yields a higher $E(AGR_{FR})$ than the benchmark strategy according to our simulation. Especially allocations with a high share invested in bonds seem to be attractive for pension funds with low risk tolerance. For instance allocations 2 to 4 bear a rather small risk, but still yield an $E(AGR_{FR})$ of 0.6% or more, equal to the minimum return necessary to maintain the current funding ratio in the long-run. In addition, implementing a Reverso strategy does not require rebalancing during the lifetime of the product. This positively effects transaction costs. However, the Reverso strategy is sensitive to the stocks selected for the underlying Reverso basket. The 12 equally weighted stocks in the Reverso basket show a slightly better risk-return relation than the international stock portfolio (MSCI World) in the reference period. A simulation for the Reverso strategy with a basket consisting of the MSCI World yields about 0.2% to 0.3% lower $E(AGR_{FR})$ for any given risk than the Reverso (basket), as shown in Figure 11.

Table 11: Asset allocation, expected annual growth rate of funding ratio and expected maximum shortfall of the Reverso (basket) strategy.

Allocation	Stocks	Bonds	Reverso	$E(AGR_{FR})$	$E(SF_{\max})$
1	0%	90%	10%	0.37%	0.76%
2	5%	85%	10%	0.59%	1.04%
3	10%	80%	10%	0.79%	1.47%
4	15%	75%	10%	0.98%	2.01%
5	20%	70%	10%	1.16%	2.66%
6	25%	65%	10%	1.33%	3.40%
7	30%	60%	10%	1.49%	4.21%
8	35%	55%	10%	1.64%	5.08%
9	40%	50%	10%	1.78%	6.00%
10	45%	45%	10%	1.91%	6.95%
11	50%	40%	10%	2.04%	7.93%
12	55%	35%	10%	2.16%	8.93%
13	60%	30%	10%	2.27%	9.95%
14	65%	25%	10%	2.38%	10.98%
15	70%	20%	10%	2.48%	12.03%
16	75%	15%	10%	2.57%	13.09%
17	80%	10%	10%	2.66%	14.15%
18	85%	5%	10%	2.74%	15.23%
19	90%	0%	10%	2.82%	16.31%

Another parameter critical for the assessment of the Reverso is the value of the product at time 0. To ensure comparability with other dynamic strategies we determined the price of one Reverso unit by a Monte Carlo simulation within our modeling framework. This price estimate does not necessarily need to be consistent with the market prices quoted by Société Générale. Based on a more sophisticated valuation model, different input parameters (e.g., stochastic volatility) and existing trading positions, Société Générale has indicated to us that they are able to offer a Reverso unit for a slightly lower premium than our price estimate. A simulation based on this quoted price yields a higher $E(AGR_{FR})$ for any given risk compared to the Reverso (basket) strategy based on the estimated price. However, we have to take into account that the Reverso product needs to be priced consistently with the parameters used for the benchmark strategy and the other dynamic strategies to ensure full comparability. Therefore, we will use the simulation results for the Reverso strategy based on the MSCI World basket in the following to ensure comparability to the other investment strategies.

Table 12: Asset allocation, expected annual growth rate of funding ratio and expected maximum shortfall of the Reverso strategy with a basket consisting of the MSCI World.

Allocation	Stocks	Bonds	Reverso	$E(AGR_{FR})$	$E(SF_{\max})$
1	0%	90%	10%	0.10%	0.95%
2	5%	85%	10%	0.33%	1.31%
3	10%	80%	10%	0.54%	1.80%
4	15%	75%	10%	0.73%	2.40%
5	20%	70%	10%	0.91%	3.09%
6	25%	65%	10%	1.09%	3.87%
7	30%	60%	10%	1.25%	4.71%
8	35%	55%	10%	1.40%	5.61%
9	40%	50%	10%	1.55%	6.55%
10	45%	45%	10%	1.68%	7.52%
11	50%	40%	10%	1.81%	8.51%
12	55%	35%	10%	1.94%	9.52%
13	60%	30%	10%	2.05%	10.55%
14	65%	25%	10%	2.16%	11.59%
15	70%	20%	10%	2.26%	12.64%
16	75%	15%	10%	2.36%	13.71%
17	80%	10%	10%	2.45%	14.78%
18	85%	5%	10%	2.53%	15.86%
19	90%	0%	10%	2.61%	16.95%

Summarizing, the risk-return spectrum of the Reverso strategy seems to be more attractive than the spectrum offered by a buy-and-hold strategy. However, the performance of the strategy strongly depends on stock selection concerning the underlying basket as well as on the market price offered by the investment bank.

5.4 Comparison of dynamic strategies

The simulations show that portfolio insurance, CPPI and the Reverso strategy have the potential to offer a more attractive risk-return spectrum than the buy-and-hold benchmark strategy. Figure 12 shows that for any given risk of the benchmark strategy, there is a dynamic strategy that yields a higher expected growth rate of the funding ratio in our modeling framework.

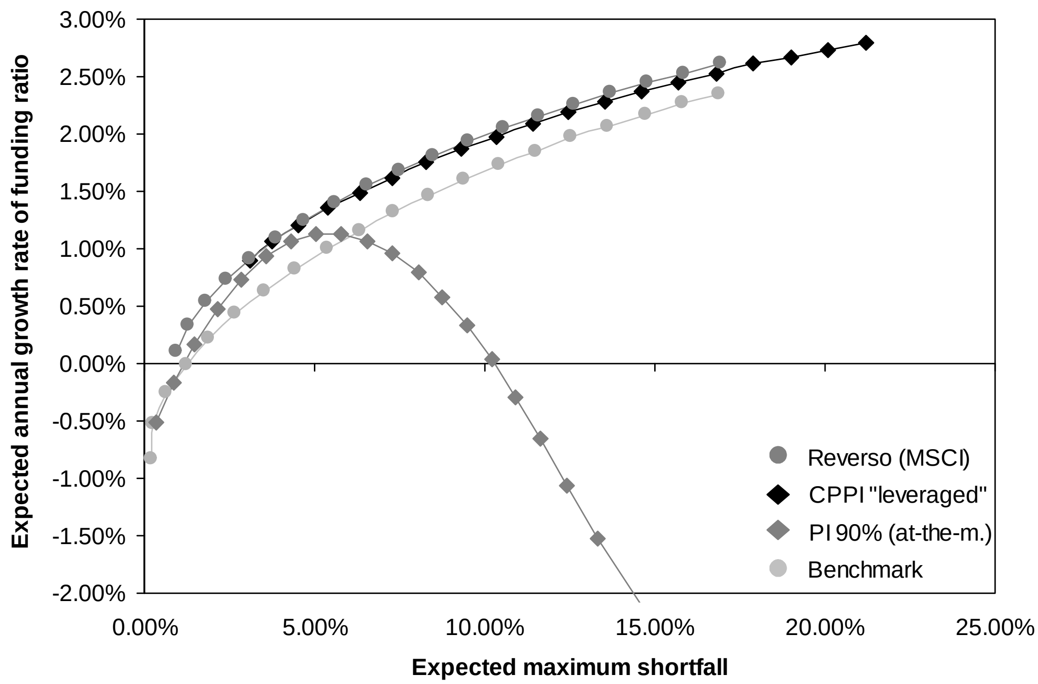

Figure 12: Expected annual growth rate of funding ratio and expected maximum shortfall of portfolio insurance with at-the-money calls and floor amount invested in long-term bonds, Reverso (MSCI) and CPPI "leveraged" compared to a buy-and-hold strategy.

In our modeling framework, portfolio insurance with a 90%-floor, at-the-money call options and the floor amount invested in long-term bonds seems to be dominated both by the Reverso strategy and CPPI "leveraged". In particular for pension funds with low risk tolerance, the Reverso strategy seems to be a valid alternative to a buy-and-hold strategy. For pension funds with a higher risk tolerance, the Reverso strategy and CPPI "leveraged" are alternatives to a buy-and-hold strategy according to our simulation.

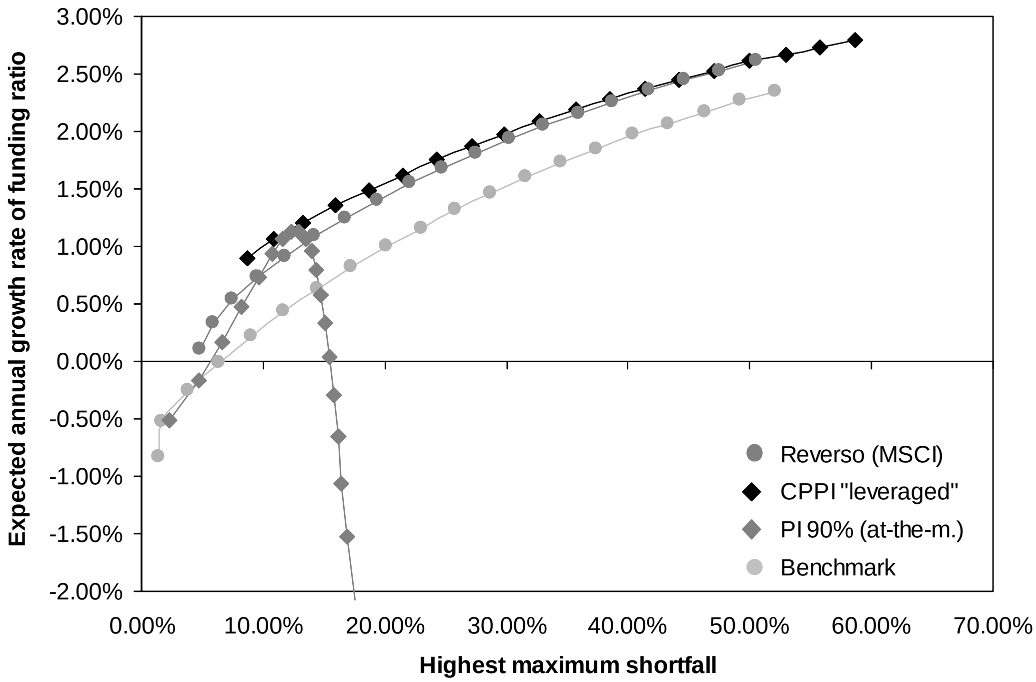

Figure 13: Expected annual growth rate of funding ratio and highest maximum shortfall ($\alpha = 5\%$) expected maximum shortfall of portfolio insurance with 90%-floor, at-the-money calls and the floor amount invested in long-term bonds, Reverso (MSCI) and CPPI "leveraged" compared to a buy-and-hold strategy.

Looking at extreme shortfall occurrences by applying a 95% quantile to the maximum shortfall distribution (highest maximum shortfall), portfolio insurance and CPPI "leveraged" slightly improve relative to the Reverso strategy. As shown in Figure 13, the Reverso strategy and CPPI "leveraged" are still very close to each other. Under the alternative risk measure, portfolio insurance strategy with a 90%-floor, at-the-money call options, and the floor invested in long-term bonds seems to be an attractive alternative for pension funds with low risk tolerance. Because the

Reverso strategy, CPPI "leveraged", as well as the buy-and-hold strategy, require large stock holdings to achieve high expected growth rates of funding ratio, highest maximum shortfalls take quite impressive values for these investment strategies. This gives an idea of what could have happened if the stock markets had not recovered after the year 2003.

5.5 Sensitivity analysis

In our modeling framework, input parameters such as expected returns and volatilities of stocks and bonds are assumed to be constant for the simulation period of 10 years. All the model input parameters are based on the reference period 1994 to 2005. To get a better understanding how sensitive our results are to adverse changes of input parameter values, the simulations are repeated for different parameter settings. The focus is on the parameters determining the dynamics of stock prices, expected stock return and volatility of stock returns, whereas input parameters relevant for the bond price process remain unchanged.

Figure 14 shows the results of a simulation with volatility of stock returns assumed to be 10% higher than in the reference period of 1994 to 2005. The expected stock return remains unchanged. Compared to the buy-and-hold strategy, CPPI "leveraged" and portfolio insurance still offer a more attractive risk-return spectrum in our modeling framework. However, a higher volatility of stock returns seems to reduce the attractiveness of the two dynamic strategies relative to the benchmark strategy. A higher volatility causes option prices to be higher. Thus, the upside potential of portfolio insurance strategy with call options is reduced because less options can be bought. On the other hand, the floor provided by a portfolio insurance strategy is not affected. In contrast, the floor under a CPPI "leveraged" strategy is negatively affected when a higher stock market volatility is assumed. The reason is the higher probability of large stock market drops immediately before rebalancing so that the floor is endangered more often. As discussed in Section 3, a Reverso strategy does well in a market environment of high and rising volatilities. Although a higher volatility of stock returns increases the initial price of the Reverso product, the larger fluctuations in value of the underlying basket in either direction yield high maximum and low minimum prices and therefore higher Reverso payoffs. Consequently, the Reverso strategy still offers an attractive risk-return spectrum compared to the benchmark strategy if a higher stock market volatility is assumed.

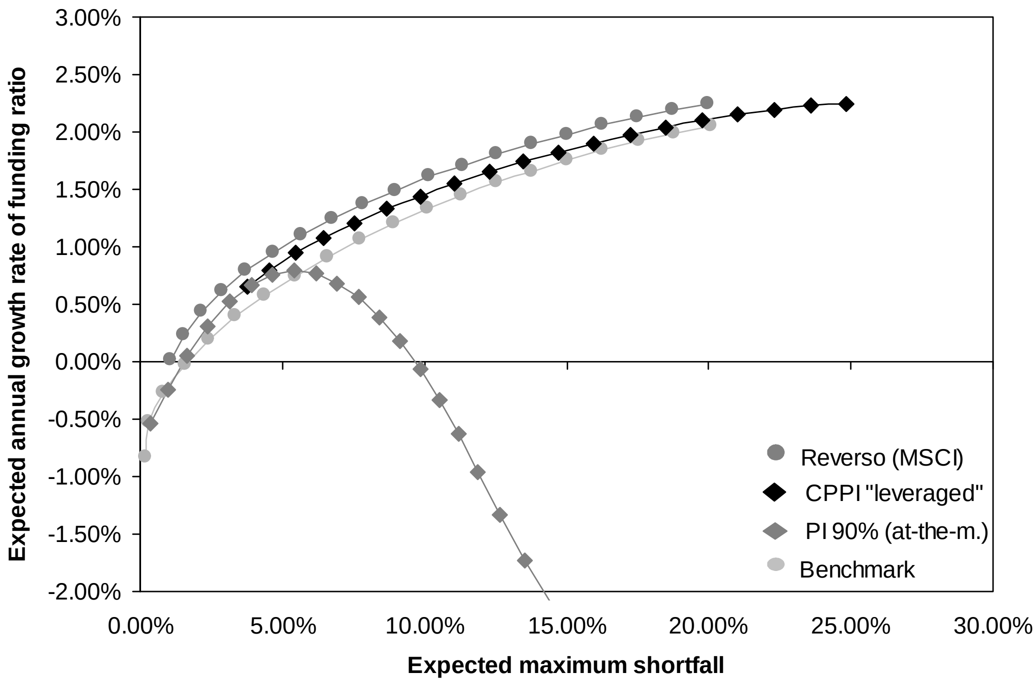

Figure 14: Expected annual growth rate of funding ratio and expected maximum shortfall of portfolio insurance with at-the-money calls and floor amount invested in long-term bonds, Reverso (MSCI) and CPPI "leveraged" compared to a buy-and-hold strategy. A 10 percent higher volatility of stock returns is assumed for the simulation.

The second input parameter determining the dynamics of stock prices is the expected stock return. Figure 15 shows the results of a simulation based on an expected stock market return 10% lower than in the reference period from 1994 to 2005. The volatility of the stock returns remains unchanged and corresponds to the volatility observed in the reference period. Again, the attractiveness of CPPI "leveraged" and portfolio insurance relative to the benchmark strategy is slightly reduced. This can be attributed to the reduced upside potential of both the leveraged stock position and the call option position. In contrast, the Reverso strategy almost maintains its advantage compared to the buy-and-hold strategy if a lower expected stock market is assumed. A lower expected stock return does not only yield lower maximum prices reducing the attractiveness of the Reverso strategy, but also lower minimum prices increasing the attractiveness of the dynamic strategy. Consequently the negative effect of lower expected stock returns on the Reverso strategy is smaller than the effect on the benchmark strategy and the other dynamic strategies.

Summarizing, simulation results for the Reverso strategy are more robust to increases in volatility and decreases in expected stock return than the simulation results obtained for CPPI and portfolio insurance.

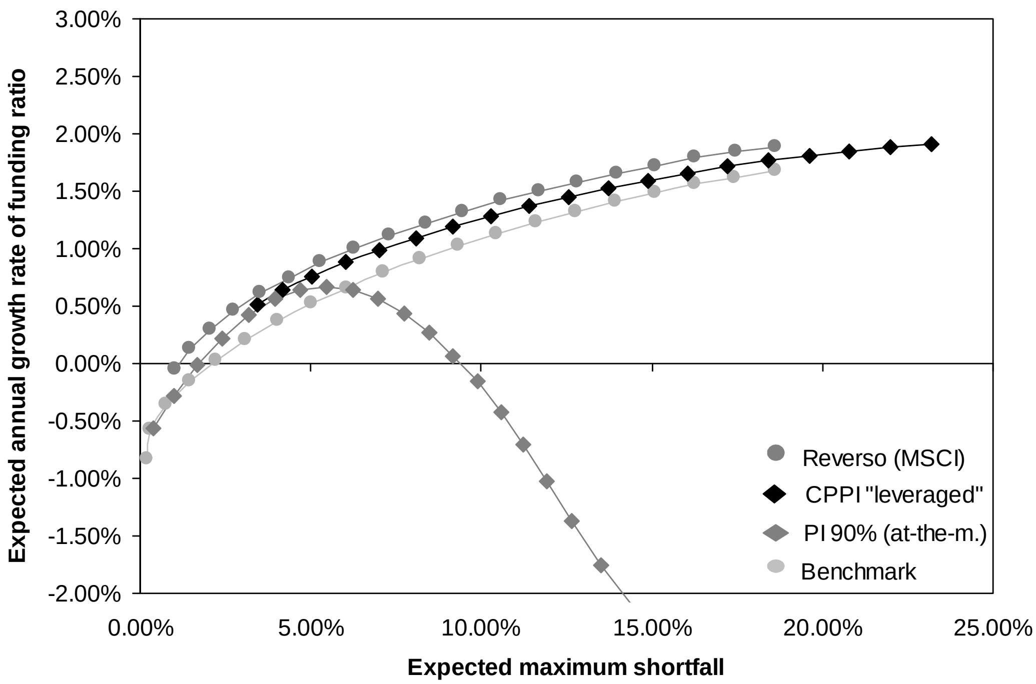

Figure 15: Expected annual growth rate of funding ratio and expected maximum shortfall of portfolio insurance with at-the-money calls and floor amount invested in long-term bonds, Reverso (MSCI) and CPPI "leveraged" compared to a buy-and-hold strategy. A 10 lower expected stock return is assumed for the simulation.

6 Conclusion

Swiss pension funds have to guarantee a minimum return on a yearly basis and simultaneously aim at maximizing the benefits of their members by generating returns persistently above the guaranteed rate. To be able to comply with both targets, financial theory suggests a dynamic investment strategy.

Because the Swiss regulator tolerates a funding ratio below 100% temporarily and no shareholders worry about their equity, unlike in the case of insurers, Swiss pension funds are able to bear relatively high risks even in absence of reserves and even if under-funded. Hence, Swiss pension funds did not actively reduce investment risks even when the majority of funds had funding ratios near or below 100%, such a policy implied that the risk associated with the investments was ultimately transferred to the members of the fund.

We show in this study that dynamic strategies have the potential to offer an attractive risk-return spectrum and might therefore be considered as an alternative to the static strategies Swiss pension funds have followed in the past.

Theoretically, some of the dynamic strategies analyzed in this study are able to reduce the shortfall risk to zero. However, under the current Swiss pension fund regulation, these zero-risk dynamic strategies only work as long as pension funds are over-funded. Consequently, even if one of these zero-risk dynamic strategies is implemented, Swiss pension funds are not able to guarantee the minimum return with 100% certainty.

When choosing a dynamic strategy, pension funds need to take into account criteria in addition to risk and return, such as transactions costs. In order to be able to fully leverage the advantages of dynamic strategies, a disciplined implementation approach, strictly based on the investment decision rules of the strategies, is crucial. Moreover, a static strategy that is "converted" into a dynamic strategy when financial markets are tumbling does not contribute to an improvement of the stability of the pension system.

Concluding, Swiss pension funds should analyze whether they can benefit from implementing dynamic investment strategies. However, dynamic strategies should not be seen as panacea for the current problems faced by many Swiss pension funds. Long-term stability of the Swiss pension fund system cannot be ensured by an intelligent choice of investment strategies, but requires modifications to the regulatory framework.

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